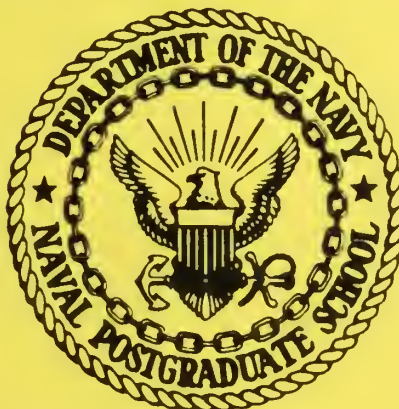


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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



MODELS OF CONFLICT,  
WITH EXPLICIT REPRESENTATION  
OF COMMAND AND CONTROL  
CAPABILITIES AND VULNERABILITIES

by

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## EXECUTIVE SUMMARY

This report describes dynamic combat models that reflect the effect of information flows together with attrition capability upon combat progress and outcome. Command and Control assets for each participant are modeled as endowed with the capacity to guide combat;  $C^2$  is also vulnerable in that it may be deliberately targetted and reduced in effectiveness. Physical attrition is modeled first by a deterministic rate process (Lanchesterian in nature), secondly by a stochastic process related to the first by ideas related to those of stochastic difference and differential equations.

The models are best exercised and explored on an interactive computer display. A FORTRAN program exists for this purpose, with displays of hypothetical "historical" combat outcomes now appearing in tabular form. Graphical displays will be provided in future work.

It seems likely that this model simulation can be developed into a gaming tool, very conveniently playable by two persons who can elect various strategies for force allocation, play the game, and learn from the results. An elaboration of the model may serve as a means for assessing the importance of increased effectiveness of equipment, either with respect to firing rate and accuracy or time for information flow.





# MODELS OF CONFLICT, WITH EXPLICIT REPRESENTATION OF COMMAND AND CONTROL CAPABILITIES AND VULNERABILITIES

Donald P. Gaver

## 1. INTRODUCTION

Many, if not most, conflicts between opposing forces R and B are conducted under some form of Command, Control and Communications ( $C^3$ ) establishment supervision. Yet few, if any, simple analytical models seem to attempt portrayal of the relationship between  $C^3$  and combat effectiveness. This report suggests models involving  $C^3$  capabilities and vulnerabilities, and indicates the manner in which the models suggested may be utilized in a gaming context.

Models constructed in the present manner were proposed by Gaver and Tonguç (1979). In that study the opposing forces were each split into two groups characterized by their respective information states: those "in the know," or capable of engaging in efficient attrition activities, and those "in the dark," and capable only of less appropriate action, or none at all. It was illustrated by Tonguç (1980) that a capability for quick transition from one information state to another could sometimes outbalance raw physical capability, such as firing rate and single-shot kill probability, thus acting as a "force multiplier."

Once the potential of the  $C^3$  component is recognized, the latter also becomes a potential target. It is, therefore, of interest to incorporate the  $C^3$  component explicitly into attrition-type models, and then to exercise the models so as to expose vulnerabilities and possibly suggest sensible doctrine. We make a stab at this program here in a highly simplified manner, feeling that informative elaborations may well be suggested after an initial look. The idea is to stray in a gingerly fashion into the area between classical Lanchester combat theory and the extensive and elaborate terrain of the modern wargame.

## 2. MODELS OF POSITION DEFENSE

### 2.1. Model I: Defense of a Stronghold (or Bastion or Beachhead)

An initial model for this situation was presented by Gaver and Tonguç (1979); see Tonguç (1979); henceforth call this the GT Model. Suppose an R-force of size  $R$  attacks a bastion (e.g. beachhead or defended position) held by  $B$ . The significance is that  $B$  is advantageously located, and in GT was assumed to suffer no casualties initially, while  $R$  is exposed and vulnerable and can only succeed by (a) surprise, or (b)  $B$ 's inability to critically diminish  $R$  before being overrun. Suppose that  $R$ 's speed of advance is (nearly) constant, and that  $R$  wins if  $R(t_0) \geq kB$ , where  $R(t_0)$  is  $R$ 's force size when the stronghold is reached at time  $t_0$ ;  $k$  represents  $R$ 's necessary advantage over  $B$  at final stages or "hand-to-hand" in order to win.

In the GT model, (and here) it was (and is) assumed that  $B$ 's divide into two combat groups: one in number  $B_u(t)$ , e.g. the number of those able to fire in an ineffective, specifically unaimed, manner, and another of size  $B_a(t)$ , e.g. the number of those capable of firing in a more effective, specifically aimed way. All sorts of refinements are possible, but for the present two information states are sufficient.

In GT it was assumed that dynamic transition between  $B_u(t)$  and  $B_a(t)$  occurred: the rate of transition  $B_u(t) \rightarrow B_a(t)$  measured the power of the  $C^3$  system. However, no attempt was made to represent that system as an explicit entity, itself being vulnerable and hence an inviting target. In the present model assume that the  $C^3$  assets of  $B$  are vulnerable to  $R$ , and that the rate of transition from  $B_u(t)$  to  $B_a(t)$  is made possible by the  $C^3$  force  $B_c(t)$ ; the latter's effectiveness can in turn be effected by  $R$ 's actions. In other words,  $R$  can attempt to, or inadvertently, target  $B_c(t)$  -- and only  $B_c(t)$  in the present model -- using the force  $R_{cc}(t)$  assigned for that purpose. To the extent that



$R_{cc}(t)$ , called the Counter- $C^3$  ( $C-C^3$ ) Force, is effective, the  $C^3$  capability of B, namely  $B_c(t)$ , is reduced by temporary suppression or outright destruction. Such reduction in turn adversely affects the quality of B's response to R's attack. Since in this model the effort (in terms of force size) allocated to  $C-C^3$  activity is removed from the Red active list, thus diminishing  $R_A$  and hence the number available to encounter B once the bastion is reached, there is clearly a trade-off opportunity for R. Too large an active force  $R_A$  at the expense of  $C-C^3$ ,  $R_{cc}$ , allows extensive attrition by B on  $R_A$  free of charge:  $B_c(t)$  may work at full effectiveness. On the other hand, too large a  $C-C^3$  force obviously penalizes the attacking force,  $R_A$ . Similar choices exist for B. Only by setting down a quantitative representation of the combat dynamics and studying its implications numerically can informed intuition be developed that may lead to a wise trade-off.

### Lanchester-Style Equations

Here are some specific Lanchester-style differential equations that represent the dynamics described above in words.

- Blue's  $C^3$  Interactions; Red's Counter- $C^3$ .

$$\frac{dB_u(t)}{dt} = - C_{ua}(B_c(t), B_u(t)) \quad (2.1)$$

$$\frac{dB_a(t)}{dt} = C_{ua}(B_c(t), B_u(t)) \quad (2.2)$$

$$\frac{dB_c(t)}{dt} = C_{cc}(B_c(t), R_{cc}(t)) \quad (2.3)$$

These equations represent the rate of change of the B-force segments with time. They describe only  $C^3$ -related activities, since physical change (attrition) of  $B_u$  and  $B_a$  has not been allowed in the present scenario. Expression (2.1) states that the rate of change reduction of the Blue ineffective (unaimed) force depends

through the function  $C_{ua}(. , .)$  upon the capacity of the Blue  $C^3$  activity, measured by  $B_c(t)$ , and upon the number of ineffective forces,  $B_u(t)$ , awaiting conversion to the effective state. Notice that this expression is left general;  $C_{ua}(. , .)$  can be specified at will, and must represent the general features of the  $C^3$  activity, including sensor performance and output analysis as well as communication. One simple, tentative, but specific representation might be

$$C_{ua}(B_c(t), B_u(t)) = \theta_{ua} B_c(t) B_u(t), \quad (2.1,a)$$

$\theta_{ua}$  being a positive constant. This expresses the appealing intuition that the rate of transfer of  $B_u$  to  $B_a$  should increase jointly with the  $C^3$  capability,  $B_c$ , and the number available for change,  $B_u$ . On the other hand, (2.1,a) does not reflect processing constraints: if each  $C^3$ -equivalent  $B_c$  - unit can service one  $B_u$  - unit at a time, then the appropriate function should resemble

$$C_{ua}(B_c(t), B_u(t)) = \theta_{ua} \min(K B_c(t), B_u(t)); \quad (2.1,b)$$

here  $K$  represents the conversion factor that allows combat units (e.g. tanks) to be interchanged for sensor-communication units, or "channels" for short. Other forms for the conversion rate  $C_{ua}$  may be derived, possibly by modeling this operational segment in the light of empirical study of any data that happens to be available.

Together, the two equations (2.1) and (2.2) simply state that a decrease in  $B_u(t)$  translates into an increase in  $B_a(t)$  during the time period  $(t, t+dt)$ . For simplicity, there is no attrition of  $B$  by  $R$ , except for that allowed to deplete  $B$ 's  $C^3$  capability,  $B_c(t)$ .

The equation (2.3) states that the rate of decrease of the Blue  $C^3$  force,  $B_c$ , depends upon the magnitude of Red Counter- $C^3$  activity,  $R_{cc}(t)$ , as well as

Blue's  $C^3$  activity,  $B_C(t)$ . Tentatively illustrate by the simple relationship

$$C_{CC}(R_{CC}(t)) = -\phi \cdot R_{CC}(t), \quad (2.3,a)$$

$\phi$  being a constant. This is a conventional Lanchester aimed-fire model. Another similar model might better reflect R's lack of knowledge of  $B_C$  location:

$$C_{CC}(R_{CC}(t)) = -\phi \cdot \left( \frac{B_C(t)}{B_0} \right) \cdot R_{CC}(t) \quad (2.3,b)$$

where  $B_0$  is proportional to the area in which  $B_C$  is concentrated, and on which  $R_{CC}$  concentrates activity. This is essentially an unaimed fire model of classical Lanchester vintage. There may be reason to transition from (2.3,b) to (2.3,a) during the course of the engagement, as information about Blue  $C^3$  increases.

- Blue and Red Physical Attrition.

$$\frac{dR_{CC}}{dt} = -\alpha_{CC}(R_{CC}(t), B_u(t), B_a(t)) \quad (2.4)$$

$$\frac{dR_A}{dt} = -\alpha_A(R_A(t), B_u(t), B_a(t)) \quad (2.5)$$

The rate functions  $\alpha_{CC}(\cdot)$  and  $\alpha_A(\cdot)$  represent the physical attrition exacted by R on the two identified B-force components: the C-C<sup>3</sup> force  $R_{CC}$ , and the active attacking force  $R_A$ . It is the magnitude of  $R_A(t_0)$ , i.e. the attack force survivorship at the time  $t_0$  when the bastion is reached, that determines whether the bastion is actually taken. However, the size effectiveness and vulnerability of  $R_{CC}$  can indirectly but decisively influence the latter variable and hence the conflict outcome. The interplay of the variables described by equations (2.1) - (2.5) quantifies the qualitative system behaviour.

Here are some specific expressions for physical attrition

$$\alpha_{cc}(R_{cc}(t), B_u(t), B_a(t)) =$$

$$\rho_{u,cc} \cdot \left( \frac{R_{cc}(t)}{R_0} \right) B_u(t) + \rho_{a,cc} \cdot \left[ \frac{k R_{cc}(t)}{k R_{cc}(t) + R_A(t)} \right] B_a(t) \quad (2.4,a)$$

The first right-hand-side (rhs) term represents the rate of attrition of  $R_{cc}$  by unaimed B fire; the parameter  $\rho_{u,cc}$  is composed of both  $B_u$ 's fire rate and single-shot kill probability. The second rhs term represents the rate of attrition of  $R_{cc}$  by aimed B fire; the parameter  $\rho_{a,cc}$  summarizes the joint effect of  $B_a$  firing rate and kill probability. The ratio  $k R_{cc} / (k R_{cc} + R_A)$  expresses the fraction of the aimed fire that is directed at the  $R_{cc}$  force, where  $k$  is a parameter that may be adjusted to account for various combat-related effects, for example: the relative hardness or invulnerability of the  $R_{cc}$  force segment as compared to the  $R_A$  force segment. It also accounts for the relative exposures of the two forces ( $R_{cc}$  and  $R_A$ ) to aimed fire by  $B_a$ . Note that if  $k = 1$  the probability that a unit of aimed fire will be directed at the  $R_{cc}$  element is simply  $R_{cc} / (R_{cc} + R_A)$  -- the fraction of the existing R force at time  $t$  that is devoted to C-C<sup>3</sup>. If  $k = 0$ , no aimed shots are directed at the C-C<sup>3</sup> element, instead being concentrated on the attackers,  $R_A$ , while if  $k \rightarrow \infty$  B aimed fire is concentrated on  $R_{cc}$ , the C-C<sup>3</sup> element. In short, the simple parameter  $k$  expresses the capabilities, vulnerabilities, and priorities of both B and R. It, or a more elaborately developed counterpart, represents combat decision choice, and would appear to have important influence on the progress of the combat.

The rate of decrease of the attack force may be specifically expressed as follows:



$$\alpha_A(R_A(t), B_u(t), B_a(t)) =$$

$$\rho_{uA} \cdot \left[ \frac{R_A(t)}{R_0} \right] B_u(t) + \rho_{aA} \cdot \left[ \frac{R_A(t)}{k R_{CC}(t) + R_A(t)} \right] B_a(t) \quad (2.5,a)$$

Again the first rhs term represents the attrition rate component resulting from the relatively ineffective (unaimed) B fire. The second rhs component represents the attrition rate from aimed fire by B. The fraction  $R_A/(k R_{CC} + R_A)$  is simply the complement of that appearing in (2.4,a); it expresses in the simplest way a composition of the relative vulnerabilities of R components and the priorities of B.

There appears to be little hope of obtaining insights directly from the differential equations (2.1) - (2.5). Of course such equations can be solved numerically, as was done for the GT model by Tonguç (1979). But another, somewhat simpler, alternative is to express the functions directly in discrete time, taking the time steps to be of unit size; perhaps 0.25 hr. might be appropriate for a start.

### Lanchester-Style Equations in Discrete Time

- Blue's  $C^3$  and Physical Interactions; Red's C-C<sup>3</sup>

$$B_u(t+1) = B_u(t) - C_{ua}(B_c(t), B_u(t)) \quad (2.6)$$

$$B_a(t+1) = B_a(t) + C_{ua}(B_c(t), B_u(t)) \quad (2.7)$$

$$B_c(t+1) = B_c(t) + C_{cc}(B_c(t), R_{cc}(t)) \quad (2.8)$$

These are easily seen to be the counterparts of (2.1) - (2.3). They may be solved recursively, starting with the initial conditions specified by  $B_u(0)$ ,  $B_a(0)$ , and  $B_c(0)$ . Choice of the initial condition by B constitutes a decision, for if initial force size is  $B = 100$ , then choosing  $B_c(0) = 20$  leaving  $B_u(0) = 80$  and  $B_a(0) = 0$  implies more faith by B in his  $C^3$  capabilities and invulnerability than does the choice  $B_c(0) = 50$  leaving  $B_u(0) = 50$  with  $B_a(0) = 0$ . The same considerations hold true for R also.



- Red's Physical Attrition

$$R_{CC}(t + 1) = R_{CC}(t) - \alpha_{CC}(R_{CC}(t), B_u(t), B_a(t)) \quad (2.9)$$

$$R_A(t + 1) = R_A(t) - \alpha_A(R_A(t), B_u(t), B_a(t)) \quad (2.10)$$

Here again one starts with initial conditions  $R_{CC}(0)$  and  $R_A(0)$  and solves recursively to find the Red attacker force size at  $t_0$ , when the defended position is reached. If R has considerable faith in the unit effectiveness of its C-C<sup>3</sup>, then presumably  $R_{CC}(0)$  is chosen to be relatively small, permitting the majority of its resources to be allocated to the attack force  $R_A(0)$ .

### 3. NUMERICAL ILLUSTRATIONS OF MODEL I PERFORMANCE

The present version of Model I is as simple as seems consistent with our attempts to blend elements of Command and Control with combat interactions between forces. Even so five state variables are needed to describe system behavior, and their inter-related evolution in time is sufficiently complex to make a direct mathematical discussion appear unprofitable. As an alternative we have elected to create a computer program that produces the numerical sequences of values assumed by the various forces as combat progresses. The computer program is written in FORTRAN; a listing appears in an Appendix. If combat is terminated at a particular time point,  $t_0$ , the relationship of  $R_A(t)$  to  $B_A(t)$  will be assumed to determine the outcome. The latter relationship is itself influenced by decision variables on each side. Here are some options and constraints for the combatants.

- Initial Red attacker force size,  $R(0)$ , and its division into attackers,  $R_A(0)$ ; and Counter- $C^3$  ( $C-C^3$ ),  $R_{CC}(0)$ ; then

$$R(0) = R_A(0) + R_{CC}(0)$$

- Initial Blue defender force size,  $B(0)$ , and its disposition into Uneffectives/Undesignated,  $B_U(0)$ ; Effectives,  $B_a(0)$ ; and  $C^3$  Forces,  $B_C(0)$ ;

$$B(0) = B_U(0) + B_a(0) + B_C(0)$$

Often,  $B_a(0) \simeq 0$  will be reasonable as an initial condition; this would represent surprise by the Red force.

Note that Blue has a technological constraint that limits its  $C^3$  capacity. Two parameters actually play this role: (i)  $\theta_{ua}$ , representing the rate per unit time of acquisition and transfer, and thus of converting  $B_U$ 's into  $B_a$ 's, and (ii)  $K$ , the capacity factor according to which force units ( $B_C(t)$  = tank equivalents, say) are made equivalent to  $C^3$  units

(sensor-communication combinations or "channels" for short); see (2.1,b) for the rate expression actually used. Initially imagine Blue to be merely endowed with these parameters, and allow them to remain fixed. It may be reasonable for them to change as combat progresses and sensors are disabled, etc. The current computer program can be straightforwardly altered to reflect such combat related damage or degradation.

Blue is assumed to be in possession of the decision parameter,  $k$ , see (2.4,a) and (2.5,a): increases in  $k$  directs a greater proportion of  $B_a$ 's fire at  $R_{CC}$  forces, while reduction of  $k$  concentrates  $B_a$  fire at the Red attacking force,  $R_A$ ;  $k = 0$  means exclusive concentration on  $R_A$ . Although the present model explicitly makes  $k$  a constant throughout the combat period, no such restriction need be at all permanent: it may perhaps be best for  $B$  to switch from  $k = \infty$  at early stages of combat -- thus maximally reducing  $R_{CC}$ 's interference with initial rapid buildup of the  $B_a$ 's force -- finally switching to  $k = 0$  later on so as to concentrate on decimating the  $R_A$  force before the defended position is reached. If for example,  $B_c(t)$  is (i) hardly reduced at all after a few periods, or (ii) is almost wiped out, then there would seem to be little reason for  $B_a$  to target  $R_{CC}$  units any longer. Of course if  $B_u$  forces are almost entirely converted to  $B_a$ 's there would again seem to be little reason to target  $R_{CC}$ , for  $B_c$ 's function has been accomplished -- at least so far as the current engagement is concerned.

Switching between two extreme values of  $k$  is reminiscent of the "bang-bang" policies of control "optimal," but there is no measure of effectiveness or figure of merit yet specified for  $B$ .

## Numerical Cases

Here are the parameter values selected for initial exercise of the model

$$R_A(0) = 120 , \quad R_{CC}(0) = 30$$

$$\phi, (\text{attrition rate of } B_C \text{ by } R_{CC}) , = 1 .$$

$$B_a(0) = 0 , \quad B_u(0) = 80 , \quad B_c(0) = 20$$

$$\rho_{uA} = \rho_{u,cc} = 0$$

For simplicity we are assuming that the Unaffected Blues have no combat performance capability, but are merely a pool of assets from which Effectives are created by the information from the central  $C^3$  facility. This assumption can easily be modified if desired.

Note carefully that in all that follows the numerical values for various parameters have been chosen for illustrative purposes only, and need bear no close resemblance to any actual values, which are in fact unknown to us. The purposes of the cases discussed is entirely exploratory. But of course we hope that that the suggestions and implications noted will promote interest in further work, leading to model refinements and the use of more nearly correct parameter values, if such can be agreed upon.

Case 1:  $\rho_{a,cc} = 1.0$  ,  $\rho_{aA} = 1.0$  (Blue Attrition rates)  
 $\phi = 1.0$  (Red Attrition rate on Blue  $C^3$ )

	Medium Speed Blue $C^3$		Rapid Blue $C^3$	
	Target: Attack	$C-C^3$	Attack	$C-C^3$
	$k=0.2$	$k=5$	$k=0.2$	$k=5$
	$\theta_{ua}=1$	$\theta_{ua}=1$	$\theta_{ua}=2$	$\theta_{ua}=2$
$\downarrow t$				
1	$(\infty)$	$(\infty)$	$\infty$	$\infty$
2	6.0	6.00	3.0	3.0
3	2.97	3.27	1.20	1.50
4	1.57	2.12	0.21	0.60
5	0.56	1.07	0.00	0.00
6	0.00	.06	$\downarrow$	$\downarrow$
7	$\downarrow$	0.00	$\downarrow$	$\downarrow$
8	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

Red/Blue Active Force Ratio as Combat Progresses  
(Red  $C-C^3$  Attrition Rate  $\phi = 1.0$ )

Fig. 3.1

The numbers in the above figure suggest that

- Increasing the speed  $\theta_{ua}$ , (reducing the response time) of Blue  $C^3$  activity has a profound effect upon the Red/Blue Force Ratio: e.g. at  $t = 4$  when  $k = 0$  (Blues concentrated on Red Attackers) Reds outnumber Blues by 1.6 to 1 if Blue's  $C^3$  speed is  $\theta_{ua} = 1$ , while doubling that rate to  $\theta_{ua} = 2$  under otherwise the same circumstances cuts the ratio to a Blue-favorable 0.21 to 1, i.e. by a factor of about eight.
- For the parameters considered, concentration by Blue on Red Attack forces pays off more than does concentration on the Red  $C-C^3$ : e.g. at  $t = 4$  with Rapid Blue  $C^3$  the advantage to Red changes from a force ratio of 1.6 to 2.1 with an increased concentration of fire by Blue on Red's  $C-C^3$ .



The comparative advantage of Red to Blue ( $R_A(t)/B_a(t)$ ), yielded by Blue emphasis on Red Attackers ( $k = 0.2$ ) instead of Blue emphasis on Red  $C-C^3$  ( $k = 5$ ) diminishes as Blue  $C^3$  speed  $\theta_{ua}$  decreases. The reason may be that when the crucial Blue  $C^3$  function is relatively weak or slow, it is profitable to spend more effort in its defense, at least until it has served its assigned purpose of converting  $B_u$ 's to  $B_a$ 's.

Case 2:  $\rho_{a,cc}(t) = 0.25 + 0.075t = \rho_{aA}(t)$

$\phi = 1.0$

In this case a space (and hence time) varying representation of the attrition rate is introduced: at  $t = 0$  when the Red force is far away, attrition rates on it are taken to be low, but they increase steadily with decreasing range (increasing  $t$ ).

	<u>Medium-Speed Blue <math>C^3</math></u>		<u>Rapid Blue <math>C^3</math></u>	
	<u>Attack</u>	<u><math>C-C^3</math></u>	<u>Attack</u>	<u><math>C-C^3</math></u>
	$k=0.2$	$k=5$	$k=0.2$	$k=5$
	$\theta_{ua}=1.0$	$\theta_{ua}=1.0$	$\theta_{ua}=2.0$	$\theta_{ua}=2.0$
$\downarrow t$				
1	$\infty$	$\infty$	$\infty$	$\infty$
2	6.0	6.00	3.00	3.00
3	3.35	3.44	1.58	1.60
4	2.25	2.50	0.88	1.12
5	1.37	1.82	0.23	0.60
6	0.40	0.89	0.00	0.00
7	0.00	0.00	$\downarrow$	$\downarrow$

Red/Blue Active Force Ratio as Combat Progresses

(Red  $C-C^3$  Attrition Rate  $\phi = 1.0$ )

Fig. 3.2

There is little qualitative change in the numbers obtained, as compared to those of Fig. 3.1. The change in the attrition rate function, increasing from a small to a larger value as time goes on, allows a larger number of Reds to survive longer than was true for Case 1. Again it seems profitable for Blue to expend most of its energy on Red Attackers.

$$\begin{aligned} \underline{\text{Case 3:}} \quad & \rho_{a,cc} = 0.25, \quad \rho_{aA} = 0.25 \\ & \phi = 1.0 \text{ (Red Attrition Rate vs Blue C}^3\text{)} \\ & \begin{cases} \theta_{ua} = 0.5 \text{ (Relatively Slow Blue C}^3\text{)} \\ \theta_{ua} = 1.0 \text{ (Relatively Rapid Blue C}^3\text{)} \end{cases} \end{aligned}$$

In order to describe the results the entire computer printout is now presented. Parameter values are shown across the page top; note that  $KC$  (computer printout) =  $k$  (text); the larger (smaller) this parameter becomes the greater (less) is concentration of Blue fire on Red  $C-C^3$ .

- Compare cases for which  $\theta_{ua} = 0.5$  to  $\theta_{ua} = 1.0$ : the force nation  $R_A/B_a$  for the slower system ( $\theta_{ua} = 0.5$ ) is about twice that for the faster ( $\theta_{ua} = 1.0$ ) system, no matter which firing strategy is adopted ( $k = 0.2, 5$ , or  $10$ ).
- The time period during which  $R_A/B_a$  is above unity (Reds have advantage), or there is near parity, is about 15 to 16 for  $\theta_{ua} = 0.5$  (Relatively Slow Blue  $C^3$ ); the same period is about 8 to 9 for  $\theta_{ua} = 1.00$ . After that period the  $R_A$  force is quickly wiped out, but there are wide differences in the readiness of the  $B_a$ 's : for  $k = KC = 5, 10$  the  $B_a$ 's soon reach their maximum of 80, ( $t \approx 12$ ), while at  $k = KC = 0.2$  the maximum is not reached until  $t \approx 42$ .

32      30.00      0.00      13.33      66.67      .00      0.00

	K	KC	RHOUC	RHOUA	RHOAA	RHOAC	PHI	H	THUA	S
	1.000	.200	0.000	0.000	.250	.250	1.000	1.000	1.000	0.000
	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA			
1	30.00	120.00	80.00	0.00	20.00	1.50	-1.00			
2	30.00	120.00	60.00	20.00	14.00	1.50	6.00			
3	29.76	115.24	46.00	34.00	9.80	1.44	3.39			
4	29.34	107.16	36.20	43.80	6.88	1.34	2.45			
5	28.78	96.77	29.32	50.68	4.86	1.21	1.91			
6	28.06	84.81	24.45	55.55	3.46	1.06	1.53			
7	27.20	71.79	20.99	59.01	2.49	.90	1.22			
8	26.16	58.08	18.50	61.50	1.81	.73	.94			
9	24.89	43.97	16.68	63.32	1.34	.55	.69			
10	23.28	29.75	15.34	64.66	1.01	.37	.46			
11	21.10	15.78	14.34	65.66	.77	.20	.24			
12	17.63	2.82	13.57	66.43	.61	.04	.04			
13	8.41	0.00	12.96	67.04	.50	0.00	0.00			
14	0.00	0.00	12.46	67.54	.46	0.00	0.00			
15	0.00	0.00	12.00	68.00	.46	0.00	0.00			
16	0.00	0.00	11.54	68.46	.46	0.00	0.00			
17	0.00	0.00	11.08	68.92	.46	0.00	0.00			
18	0.00	0.00	10.62	69.38	.46	0.00	0.00			
19	0.00	0.00	10.16	69.84	.46	0.00	0.00			
20	0.00	0.00	9.70	70.30	.46	0.00	0.00			
21	0.00	0.00	9.24	70.76	.46	0.00	0.00			
22	0.00	0.00	8.78	71.22	.46	0.00	0.00			
23	0.00	0.00	8.32	71.68	.46	0.00	0.00			
24	0.00	0.00	7.86	72.14	.46	0.00	0.00			
25	0.00	0.00	7.40	72.60	.46	0.00	0.00			
26	0.00	0.00	6.94	73.06	.46	0.00	0.00			
27	0.00	0.00	6.48	73.52	.46	0.00	0.00			
28	0.00	0.00	6.02	73.98	.46	0.00	0.00			
29	0.00	0.00	5.56	74.44	.46	0.00	0.00			
30	0.00	0.00	5.11	74.89	.46	0.00	0.00			
31	0.00	0.00	4.65	75.35	.46	0.00	0.00			
32	0.00	0.00	4.19	75.81	.46	0.00	0.00			
33	0.00	0.00	3.73	76.27	.46	0.00	0.00			
34	0.00	0.00	3.27	76.73	.46	0.00	0.00			
35	0.00	0.00	2.81	77.19	.46	0.00	0.00			
36	0.00	0.00	2.35	77.65	.46	0.00	0.00			
37	0.00	0.00	1.89	78.11	.46	0.00	0.00			
38	0.00	0.00	1.43	78.57	.46	0.00	0.00			
39	0.00	0.00	.97	79.03	.46	0.00	0.00			
40	0.00	0.00	.51	79.49	.46	0.00	0.00			
41	0.00	0.00	.05	79.95	.46	0.00	0.00			
42	0.00	0.00	0.00	80.00	.46	0.00	0.00			
43	0.00	0.00	0.00	80.00	.46	0.00	0.00			
44	0.00	0.00	0.00	80.00	.46	0.00	0.00			
45	0.00	0.00	0.00	80.00	.46	0.00	0.00			
46	0.00	0.00	0.00	80.00	.46	0.00	0.00			

Red/Blue Force Changes as Combat Progresses ( $\phi=1.0$ )

(K = KC = 0.2)

Fig. 3.3a

47 0.00 0.00 0.00 80.

	K 1.000	KC 5.000	RHOUC 0.000	RHOUA 0.000	RHOAA .250	RHOAC .250	FHI 1.000	H 1.000	THUA 1.000	S 0.000
	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA			
1	30.00	120.00	80.00	0.00	20.00	1.50	-1.00			
2	30.00	120.00	60.00	20.00	14.00	1.50	6.00			
3	27.22	117.78	46.00	34.00	9.80	1.47	3.46			
4	22.67	113.83	36.20	43.80	7.13	1.42	2.60			
5	17.20	108.35	29.07	50.93	5.52	1.35	2.13			
6	11.57	101.25	23.55	56.45	4.57	1.27	1.79			
7	6.44	92.27	18.99	61.01	4.04	1.15	1.51			
8	2.42	80.96	14.95	65.05	3.78	1.01	1.24			
9	.32	66.87	11.17	68.83	3.68	.84	.97			
10	0.00	50.06	7.48	72.52	3.67	.63	.69			
11	0.00	31.93	3.81	76.19	3.67	.40	.42			
12	0.00	12.89	.14	79.86	3.67	.16	.16			
13	0.00	0.00	0.00	80.00	3.67	0.00	0.00			
14	0.00	0.00	0.00	80.00	3.67	0.00	0.00			
15	0.00	0.00	0.00	80.00	3.67	0.00	0.00			
16	0.00	0.00	0.00	80.00	3.67	0.00	0.00			
17	0.00	0.00	0.00	80.00	3.67	0.00	0.00			
18	0.00	0.00	0.00	80.00	3.67	0.00	0.00			

(k = KC = 5.0)

Fig. 3.3b

	K	KC	RHOUC	RHOUA	RHOAA	RHOAC	PHI	H	THUA	S
	1.000	10.000	0.000	0.000	.250	.250	1.000	1.000	1.000	0.000
	RCC(T)	RA(T)	BU(T)	BA(T)	BC(T)	RA/BU+BA	RA/BA			
1	30.00	120.00	80.00	0.00	20.00	1.50	-1.00			
2	30.00	120.00	60.00	20.00	14.00	1.50	4.00			
3	26.43	118.57	46.00	34.00	9.80	1.48	3.49			
4	20.56	115.94	36.20	43.80	7.21	1.45	2.65			
5	13.56	111.99	28.99	51.01	5.73	1.40	2.20			
6	6.58	106.22	23.26	56.74	4.95	1.33	1.87			
7	1.15	97.46	18.31	61.69	4.63	1.22	1.58			
8	0.00	83.67	13.69	66.31	4.57	1.05	1.26			
9	0.00	67.09	9.11	70.89	4.57	.84	.95			
10	0.00	49.37	4.54	75.46	4.57	.62	.65			
11	0.00	30.50	0.00	80.00	4.57	.38	.38			
12	0.00	10.50	0.00	80.00	4.57	.13	.13			
13	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
14	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
15	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
16	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
17	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
18	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
19	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
20	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
21	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
22	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
23	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
24	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
25	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
26	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
27	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
28	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
29	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
30	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
31	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
32	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
33	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
34	0.00	0.00	0.00	80.00	4.57	0.00	0.00			
35	0.00	0.00	0.00	80.00	4.57	0.00	0.00			

(k = KC = 10)

Fig. 3.3c



Case 4:  $\rho_{a,u} = 0.10$ ,  $\rho_{aA} = 0.1$

$\phi = 0.20$ ,  $\theta_{ua} = 0.25$

$\downarrow t$	$B_u(0)=90(=B_A(\infty)), B_C(0)=10$		$B_u(0)=80(=B_A(\infty)), B_C(0)=20$		$B_u(0)=60(=B_A(\infty)), B_C(0)$	
	k: 0	10	0	10	0	10
1	48.0	48.0	24.0	24.0	12.0	12.0
2	24.7	24.7	12.3	12.4	6.1	6.2
3	16.9	17.0	8.4	8.5	4.2	4.2
4	13.0	13.1	6.4	6.5	3.2	3.3
5	10.6	10.8	5.2	5.4	2.6	2.8
10	5.74	6.05	2.62	2.92	1.54	1.92
15	3.95	4.39	1.57	1.95	0.97	1.47
20	2.91	3.43	0.88	1.22	0.46	0.97
30	1.52	1.96	0.00	0.07	0.00	0.00
40	0.44	0.71	$\downarrow$	0.00	$\downarrow$	$\downarrow$

Red/Blue Active Force Ratio as Combat Progresses

Fig. 3.4

The most striking effect visible in the table is that Blue improves his performance relative to Red by decreasing the Effectives and increasing the allocation of his forces to  $C^3$ . A nearly 3-to-1 improvement (for B) of the force ratio at  $t = 5$  is apparent. The effect of the parameter  $k$ , which dictates the fraction of B energy expended to deplete Red  $C-C^3$ , is very small.

#### 4. A STOCHASTIC VERSION OF MODEL I.

The evolutionary equations (2.1) - (2.5), or equivalently (2.6) - (2.10) are entirely deterministic, a feature that seems unrealistic since in reality uncertainty and random variability abound. There are several ways in which uncertainty may be allowed to intrude into the formulations; for instance

- (i) Through the necessity of estimating parameters ( $\rho_{u,cc}$ ,  $\rho_{a,cc}$ ,  $k$ , etc.) from data, assuming the model specification is "correct", or at least adequate.
- (ii) Through the necessity of using simplistic models such as (2.1) - (2.5) to represent a more complex reality.
- (iii) By explicitly permitting randomness to enter the dynamic equations as an additional driving force.
- (iv) Other possibilities; combinations of the above, for example.

Begin by adding a random perturbation term to the discrete time equations (2.6) - (2.10) as in (iii) above:

$$\begin{aligned}
 B_u(t+1) &= B_u(t) - C_{ua}(B_c(t), B_u(t)) + \sigma_u(\Delta W_{t+1}(u)) \\
 B_a(t+1) &= B_a(t) + C_{ua}(B_c(t), B_u(t)) + \sigma_a(\Delta W_{t+1}(a)) \\
 B_c(t+1) &= B_c(t) - C_{cc}(B_c(t), R_{cc}(t)) + \sigma_c(\Delta W_{t+1}(c)) \\
 R_{cc}(t+1) &= R_{cc}(t) - \alpha_{cc}(R_{cc}(t), B_u(t), B_a(t)) + \sigma_{cc}(\Delta W_{t+1}(cc)) \\
 R_A(t+1) &= R_A(t) - \alpha_A(R_A(t), B_u(t), B_a(t)) + \sigma_A(\Delta W_{t+1}(A))
 \end{aligned} \tag{4.1}$$

The notation  $\chi$  denotes a random function. The vector of random components  $W_t(u), W_t(a), W_t(c), W_t(cc), W_t(A)$  will be taken to be one of not necessarily independent Wiener processes, sampled at time points  $t = 1, 2, 3, \dots$ . Thus the equations (4.1) turn out to be analogous to Ito-type stochastic differential equations; such equations have been used by Lehoczky and Perla (1978) to describe combat situations.

#### 4.1. Explicit Representation of Random Terms.

By arguments analogous to those of Lehoczky and Perla (1978), or of Gaver and Lehoczky (1977) in a different context, we write down expressions for the scales  $\sigma(\cdot)$  of the random components written as  $\sigma(\Delta W_{t+1}(\cdot))$  in (4.1).

We argue heuristically that if the stochastic processes are nearly Markovian and further are superpositions of many point (e.g. birth-death) processes describing the changes of individuals states in a relatively short time period, then it is reasonable that  $\sigma(\cdot)$  be equal to the square-root of the individual drift

(deterministic) terms, the latter being given by the expression

$C_{ua}(B_c(t), B_u(t)), C_{cc}(B_{cc}(t), B_u(t), B_a(t))$ , etc. appearing on the rhs of (4.1).

The latter is suggested by the approximately Poisson nature of the changes in state of the system over relatively short time intervals. Recall that the standard deviation of a Poisson random variable (state change, here) is equal to the square-root of the mean (state change), or drift in diffusion theory jargon. Furthermore, the random elements  $\Delta W_{t+1}(n), \Delta W_{t+1}(a)$ , etc., are realizations of independent unit normal or Gaussian random variables. Some readers will recognize the resulting system to be a discrete-time version of Itô-type stochastic differential equations; see Arnold ( ) for basic information.

Here are some explicit examples, following (2.1,b), (2.3), (2.4,a), (2.5,a).

$$\begin{aligned}\sigma_u \cdot (\Delta W_{t+1}(u)) &= \left[ \sqrt{\theta_{ua} \min\{KB_c(t), B_u(t)\}} \right] (\Delta W_{t+1}(u)) \\ \sigma_a \cdot (\Delta W_{t+1}(a)) &= \left[ \sqrt{\theta_{ua} \min\{KB_c(t), B_u(t)\}} \right] (\Delta W_{t+1}(a)) ,\end{aligned}\quad (4.2)$$

where

$\Delta W_{t+1}(a) \sim N(0,1)$ , i.e. has the Normal distribution with mean 0,

and standard deviation 1,

and

$$\Delta W_{t+1}(a) = - \Delta W_{t+1}(u),$$

the latter because a fluctuation away from the mean change in one direction--down, say--for  $B_u(t)$  is exactly matched by one in the up direction for  $B_a(t)$  during time interval  $t$  to  $t+1$ . Next, using (2.3,b),

$$\sigma_c \cdot (\Delta W_{t+1}(c)) = \left[ \sqrt{\phi \cdot \left( \frac{B_c(t)}{B_0} \right) R_{cc}(t)} \right] (\Delta W_{t+1}(c)) \quad (4.3)$$

where  $\Delta W_{t+1}(c) \sim N(0,1)$ , and  $\Delta W_{t+1}(c)$  is independent of  $\Delta W_{t+1}(a)$  as well as of past values  $\Delta W_\tau(c)$ ,  $\tau = 0,1,2,\dots,t$ .

Next,

$$\begin{aligned} \sigma_{cc} \cdot (\Delta W_{t+1}(cc)) &= \left[ \sqrt{\rho_{u,cc} \cdot \left[ \frac{R_{cc}}{R_0} \right] B_u(t)} \right] (\Delta W_{t+1}(u,cc)) \\ &+ \left[ \sqrt{\rho_{a,cc} \left[ \frac{kR_{cc}(t)}{kR_{cc}(t) + R_A(t)} \right] B_a(t)} \right] (\Delta W_{t+1}(a,cc)) \end{aligned} \quad (4.4)$$

where  $\Delta W_{t+1}(cc,u)$  and  $\Delta W_{t+1}(cc,a)$  are independent random elements, both  $\sim N(0,1)$ , that represent respectively the fluctuation away from mean attrition on the Red C-C<sup>3</sup> facility caused by unaimed (ineffective) fire and by aimed (effective) fire.

Finally, use of (2.5a) provides

$$\begin{aligned} \sigma_A \cdot (\Delta W_{t+1}(A)) &= \left[ \sqrt{\rho_{uA} \left[ \frac{R_A(t)}{R_0} \right] B_u(t)} \right] (\Delta W_{t+1}(u,A)) \\ &+ \left[ \sqrt{\rho_{aA} \left[ \frac{R_A(t)}{kR_{cc}(t) + R_A(t)} \right] B_a(t)} \right] (\Delta W_{t+1}(a,A)). \end{aligned} \quad (4.5)$$

Again,  $\Delta W_{t+1}(u,A)$  and  $\Delta W_{t+1}(a,A)$  are independent and normally distributed, representing fluctuations around mean attrition on the  $R_A$  - component. Under the



present simple circumstances in which approximate independence may be justified, we can actually combine the two "noise" terms in (3.4) and (3.5) to obtain the simpler forms

$$\sigma_{cc} \cdot \Delta W_{t+1}(cc) =$$

$$= \left[ \sqrt{\rho_{u,cc} \left[ \frac{R_{cc}(t)}{R_0} \right] B_u(t) + \rho_{a,cc} \left[ \frac{k R_{cc}(t)}{k R_{cc}(t) + R_A(t)} \right] B_a(t)} \right] (\Delta W_{t+1}(cc)) \quad (4.4,a)$$

and

$$\sigma_A \cdot \Delta W_{t+1}(A) =$$

$$\left[ \sqrt{\rho_{uA} \left[ \frac{R_A(t)}{R_0} \right] B_u(t) + \rho_{aA} \left[ \frac{R_A(t)}{k R_{cc}(t) + R_A(t)} \right] B_a(t)} \right] (\Delta W_{t+1}(A)) \quad (4.5,b)$$

This will reduce simulation difficulties by making it unnecessary to sample two normal random variables for each of the last equations. The introduction of correlation is easy, if justified.

It might be pointed out that considerable freedom exists in the choice of the distribution of all of the noise terms  $\Delta W_{t+1}(\cdot)$ : they need not be normal, nor need they be independent, nor, in fact, need they be independent of the corresponding state value. Of course a random fluctuation that sends, say,  $R_A(17)$  formally negative must in truth merely wipe out the A-force, i.e. reduce it to zero. Intuition indicates that a formal (and forbidden) passage below zero for  $R_A(t+1)$ , and that the latter is unlikely to be extensive by virtue of the small noise variation near  $R_A = 0$  for the particular form of (4.5,b). However, choice of a more gaudily variable noise increment, e.g. with  $\Delta W_{t+1}(\cdot)$  now chosen to be long tailed, perhaps in a Cauchy-like manner, will likely require more extensive fixing at the boundary. This is no reason to avoid such models, for there is nothing holy about the normal (or Wiener-like) variation save for its appearance



as the noise when a basic Markov structure is assumed to underlie the present models. Actually, over-variation (from the Gaussian/normal) and serial dependence may well usefully represent mixtures of normals (or other) distributions resulting from factors such as terrain, visibility, and many other features which combine to generate departures from the systematic deterministic models analogous to the classical Lanchester forms (2.1) - (2.5).

## 5. MODEL II: TWO FORCES IN COMBAT WITH MUTUAL ATTRITION

The model described earlier dealt with a special situation in which one force (Red) was the attacker of another in a defended position (Blue). Lack of symmetry was evident. The model of this section represents an equally stylized but now symmetric situation in which both forces are capable of causing attrition on each other. Once again, guidance is furnished by explicitly represented, and vulnerable  $C^3$  agencies. Our models allow for different targetting strategies.

Notation is as follows:

$R_a(t)$  = the number of active Reds at  $t$ , meaning the number of  $R$ 's actually targetted on, and firing at, Blue units.

$B_a(t)$  = the corresponding Blue force size.

$R_i(t)$  = the number of ineffective (or inactive)  $R$ 's at  $t$ , meaning the number of  $R$ 's currently untargetted and awaiting new assignment.

$B_i(t)$  = the corresponding Blue force size.

$R_c(t)$  = the size of the Red  $C^3$  agency at  $t$ , i.e. the force responsible for switching  $R_i$  - units to  $R_a$  - units.

$B_c(t)$  = the corresponding Blue force size.

The quantities  $R_c(t)$  and  $B_c(t)$  are to be viewed as the command and control authorities responsible for the individual direct combat elements on their respective sides. The state variables  $R_a(t)$ ,  $R_i(t)$ ,  $B_a(t)$ ,  $B_i(t)$  may be thought of in units of individual tanks or ships, or as aggregations such as battalions, companies, naval task groups, or whatever is appropriate for the particular situation under consideration. The intention of the present model is to simply express changes in the respective force sizes in terms of rate processes--both information transfer rates and physical attrition rates--and in terms of initial allocations of resource

It is clear that when Red and Blue forces come into contact a variety of possible behaviors may occur. The models developed here are intended to represent the

consequences of a few simplified versions of the true complexity possible. In particular, they permit the study of different target category priority schemes.

### Red(Blue) Force State Equations

The general form of the state change equations now follows. They are given only for Red, but the Blue equations are symmetric.

$$\begin{aligned}
 R_a(t+1) = R_a^*(t) - & \underbrace{C_{BRA}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Physical Attrition, } R_a} \\
 & + \underbrace{D_{RA}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t), B_i^*(t), B_c^*(t))}_{\text{Effect of Information State Change, } R_a}
 \end{aligned} \tag{5.1}$$

$$\begin{aligned}
 R_i(t+1) = R_i^*(t) - & \underbrace{C_{BRI}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Physical Attrition, } R_i} \\
 & + \underbrace{D_{RI}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t), B_i^*(t), B_c^*(t))}_{\text{Effect of Information State Change, } R_i}
 \end{aligned} \tag{5.2}$$

$$\begin{aligned}
 R_c(t+1) = R_c^*(t) - & \underbrace{C_{BRC}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Red } C^3 \text{ Capacity Attrition and Suppression}} \\
 & + \underbrace{D_{RC}(R_a^*(t), R_i^*(t), R_c^*(t), B_a^*(t))}_{\text{Red } C^3 \text{ Capacity Restoration and Recovery}}
 \end{aligned} \tag{5.3}$$

- Additionally, to compute  $R_a^*, R_i^*, R_c^*$  from  $R_a, R_i, R_c$ , utilize the appropriate physical constraints:

$$R_a^*(t+1) = \min[R_a(0) + R_i(0), \max\{R_a(t+1), 0\}]$$

$$R_i^*(t+1) = \min[R_a(0) + R_i(0), \max\{R_i(t+1), 0\}]$$

$$R_c^*(t+1) = \min[R_c(0), \max\{R_c(t+1), 0\}]$$

- Initial conditions, i.e. values of  $R_a(0)$ ,  $R_i(0)$ ,  $R_c(0)$ , are required to start the process, after which the various force sizes are calculated recursively in time.
- Abbreviate:  $C_{BRA}(\text{arguments as in (5.1)}) \equiv C_{BRA}(t)$

$$D_{RA}(\text{arguments as in (5.1)}) \equiv D_{RA}(t),$$

etc.

- Blue State Equations resemble (5.1), (5.2), (5.3) with changes in rate definition and notation

<u>Red</u>	<u>Blue</u>
$C_{BRA}$	$C_{RBA}(B_a^*(t), B_i^*(t), B_c^*(t), R_a^*(t)), \text{ etc.}$
$D_{RA}$	$D_{BA}$
$C_{BRI}$	$C_{RBI}$
$D_{RI}$	$D_{BI}$
$C_{BRC}$	$C_{RBC}$
$D_{BRC}$	$D_{RBC}$

There now follow some specific expression for the above rates, intended to illustrate possible effort allocation strategies.

A. Greedy Allocation (Blue vs. Red)

Priority Order: Red Actives, Red Inactives, Red  $C^3$

a)  $C_{BRA}(t) = \rho_A B(R_a^*(t)),$

$B(R_a^*(t)) = \text{Number of Blue Actives vs. Red Actives}$

$$= \min[B_a^*(t), R_a^*(t)] \quad (5.4)$$

- Note that all  $B_a$ 's fire one-on-one on  $R_a$ 's if  $B_a$  force <  $R_a$  force; otherwise one-on-one until  $R_a$  targets are insufficient, leaving  $B_a^*(t) - R_a^*(t)$  to be used against  $R_i$ 's, which are next on the priority list.

b)  $C_{BRI}(t) = \rho_I B(R_i^*(t)),$

$B(R_i^*(t)) = \text{Number of Blue Actives vs. Red Inactives}$

$$= \min[\max\{B_a^*(t) - B(R_a^*(t)), 0\}, R_i^*(t)] \quad (5.5)$$

c)  $C_{BRC}(t) = \rho_C B(R_C^*(t))$

$B(R_C^*(t)) = \text{Number of Blue Actives vs. Red } C^3$

$$= \min[\max\{B_a(t) - B(R_a^*(t)) - B(R_i^*(t)), 0\}, R_C^*(t)] \quad (5.6)$$

- Change of priority is easily accomplished: if priority sequence is

x first, y second, z third



$$C_{BRx}(t) = \rho_x B(R_x^*(t)), \quad B(R_x^*(t)) = \min[B_a^*(t), R_x^*(t)]$$

$$C_{BRy}(t) = \rho_y B(R_y^*(t)), \quad B(R_y^*(t)) = \min_{R_y^*(t)} \left[ \max\{B_a^*(t) - B(R_x^*(t)), 0\}, \right. \\ \left. R_y^*(t) \right] \quad (5.7)$$

$$C_{BRz}(t) = \rho_z B(R_z^*(t)), \quad B(R_z^*(t)) = \min_{R_z^*(t)} \left[ \max\{B_a^*(t) - B(R_x^*(t)) - B(R_y^*(t)), 0\}, \right. \\ \left. R_z^*(t) \right]$$

- Same, with obvious changes, for Red on Blue
- Attrition rates of Blue, by Red, are denoted respectively by  $\beta_a, \beta_i, \beta_c$ .
- Attrition rates can be made time (space) dependent.
- Note that the attrition law is assumed for illustration only to be in accordance with a Lanchesterian linear law. Other types of laws can clearly replace this one.

#### B. Proportional Allocation (Blue vs. Red)

$$a) \quad C_{BRA}(t) = \rho_A \left[ \frac{k_A R_a^*(t)}{k_A R_a^*(t) + k_I R_i^*(t) + k_C R_c^*(t)} \right] B_a^*(t) \quad (5.8)$$

- $k_A, k_I, k_C$  are control "constants", representing target priorities. They can be changed or combat progresses, if desired. If  $k_A \rightarrow \infty$  (practically  $k_A/k_I \simeq k_A/k_C \simeq 10$  should do) there is heavy emphasis on  $R_a$ 's
- $\rho_A$  is attrition rate (firing rate times single-shot kill probability).

$$b) \quad C_{BRI}(t) = \rho_I \left[ \frac{k_I R_i^*(t)}{k_A R_a^*(t) + k_I R_i^*(t) + k_C R_c^*(t)} \right] B_a^*(t) \quad (5.9)$$

$$c) \quad C_{BRC}(t) = \rho_C \left[ \frac{k_C R_C^*(t)}{k_A R_a^*(t) + k_I R_i^*(t) + k_C R_C^*(t)} \right] B_a^*(t) \quad (5.10)$$

- Same for Red on Blue with appropriate notational changes.
  - This is an alternative strategy that allocates some  $B_a$ -effort to all components of the Red force. It samples rather than gulps.
- There are many possible alternatives.

### Red(Blue) Information State Change Equations

Here are some sample equations that are tentatively proposed to model the effects of philosophy and capability of the  $C^2$  component.

#### (1) Tight Central Control(Reds); $C^2$ Attrition

Only  $R_a$ 's may fire at Blues. When a Red firing engagement terminates, it returns to the  $R_i$  status (becomes inactive). The  $R_C$  force acts to change or return  $R_i$ 's to  $R_a$ 's. Thus

$$(a) \quad D_{RA}(t) = \underbrace{\min\{C_R K_R R_C^*(t), C_R R_i(t), R_i(t)\}}_{(R_i \rightarrow R_a \text{ rate; } C^2 \text{ process})} \quad (5.11)$$

$$- \underbrace{\left[ C_{RBA}(t) + C_{RBI}(t) + C_{RBC}(t) \right]}_{\text{(Number of Red vs Blue Engagements Terminated in } t^{\text{th}} \text{ Period, or Number of } R_a \text{'s Released to } R_i \text{'s.)}}$$

$$(b) \quad D_{RI}(t) = - D_{RA}(t) \quad (5.12)$$

$$(c) \quad D_{RC}(t) = 0. \quad (\text{No Red } C^2 \text{ recovery from suppression.}) \quad (5.13)$$

- Note that in this model both physical attrition (which is permanent) and lack of information (temporary, unless  $C^2$  is inadequate) reduce Red effectiveness.

(2) Loose Central Control;  $C^2$  Attrition

Same as (1), but retain only the first term

$$(a) \quad D_{RA}(t) = \min\{C_R K_R R_C^*(t), C_R R_i(t), R_i(t)\} \quad (5.14)$$

$$(b) \quad D_{RI}(t) = - D_{RA}(t) \quad (5.15)$$

$$(c) \quad D_{RC}(t) = 0 \quad (5.16)$$

- Note that this model assumes that once an  $R_i$  changes to  $R_a$  status it remains so until physically removed.
- A model that lies between (1) and (2) may be attractive, in which case one might multiply the second term in (5.11) by a time (distance) dependent factor less than unity to represent the fraction of the terminating Red engagements that must have recourse to central  $C^2$  re-direction.

## 6. NUMERICAL ILLUSTRATIONS OF MODEL II PERFORMANCE.

In this section we present some examples of Model II behavior. Once again these numerical results are obtained from a computer program written in FORTRAN; a listing appears in an Appendix. The intention is to provide some feeling for the sensitivity of the model to changes in parameters and combat philosophy or priorities. Careful study of the outcomes will be helpful in leading one to comprehend the complex dynamics of this model and others evolving from it.

Discussion. For convenience, consider the columns in the Figure labelled 1  $\rightarrow$  9 from the left; e.g. Col. 3 is headed  $c_R=1,(2)$ , under  $\rho_C=0.1$ , meaning that Red's  $C^2$  rate is unity, the attrition rate of Blue on Red is  $\rho_C=0.1$ , and (1) Strong Central Control is in use by both antagonists.

- Each combatant's Active component first grows, as  $C^2$  changes Inactive to Actives, and then dwindles by attrition. This is to be expected.
- Compute Col. 2 and Col. 3: Strong Central Control tends to overload  $C^2$  to the detriment of Red Actives. See also Col. 6 and Col. 7: same lesson for Blue Actives. A tradeoff would occur if S.C.C. by Red was combined with higher attrition rates for Red than Blue, Blue using L.C.C.
- Compare Col. 2 and Col. 4: increasing (doubling)  $C^2$  processing rate under S.C.C. provides a decided initial improvement for Red. but this disappears later; the number of Inactives becomes small, so there are no resources from which to draw to increase the complement of Actives.

<u>t</u>	<u>R<sub>a</sub>(t)</u>				<u>B<sub>a</sub>(t)</u>			
	$\rho_C=0.1$		$\rho_C=0.5$		$\rho_C=0.1$		$\rho_C=0.5$	
	<u>c<sub>R</sub>=1,(1)</u>	<u>c<sub>R</sub>=1,(2)</u>	<u>c<sub>R</sub>=2,(1)</u>	<u>c<sub>R</sub>=2,(2)</u>	<u>c<sub>R</sub>=1,(1)</u>	<u>c<sub>R</sub>=1,(2)</u>	<u>c<sub>R</sub>=2,(1)</u>	<u>c<sub>R</sub>=2,(2)</u>
0	0	0	0	0	0	0	0	0
1	20	20	40	40	10	10	10	10
2	37	39	75	79	18	19	18	19
3	52	57	90	97	24	27	24	27
4	75	91	83	91	34	41	34	41
6	81	87	81	87	37	47	37	47
7	77	82	77	82	40	52	40	52
8	73	77	73	77	42	57	42	57
9	69	71	69	71	43	61	44	61
10	65	65	65	65	45	<u>65</u>	45	65
15	44	38	44	38	<u>48</u>	64	<u>48</u>	52
20	26	23	26	23	44	48	32	36
25	16	13	16	13	35	38	21	26
Col:								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Fig. 6.1

$$c_B = 1.0, \quad c = 0.10$$

Priorities: Actives - Inactives -  $c^2$

(1): Strong Central Control

(2): Loose Central Control



Discussion. This Fig. 6.2 illustrates the effect of changes in target priority under different philosophies of central control.

- All else being equal, S.C.C., indicated by (1), is a handicap as illustrated by comparison of Col. 2 and 4 for Reds, and corresponding Cols. 6 and 8 for Blues. The penalty for Blue is higher possibly because of its original smaller complement of  $C^2$  assets, and hence its poorer traffic handling capability.
- Suppose Blue reverses firing priority, targetting  $C^2$  first, then  $R_a$ , finally  $R_i$ . Compare Cols. (3) and (7) to Cols. (2) and (6). Note that initially  $R_a$  actually improves when  $C^2$  is first priority because  $R_a$  is only lightly diminished and  $R_c$  remains temporarily adequate. It appears that this strategy change by B is ineffective because Red's  $C^2$  facilities are ample enough to withstand the attack.
- Suppose Red reverses firing strategy, targetting Blue  $C^2$  first, then  $B_a$ , and finally  $R_i$ . Adopt loose control, (2), for illustration. Note that the effect on  $B_a$  of Red's  $C^2$ -first strategy is initially small, but as the combat proceeds  $B_a$  force size is considerably reduced.

The last two comparisons suggest that primary attack on the opponent's  $C^2$  force is advantageous when that force is meager (or especially vulnerable), while such an attack is actually counterproductive in case the  $C^2$  force is adequate. Of course this simply suggests that high priority is best placed on attacking the weakest point in the system. In the present example weakness is merely a matter of numbers ( $C^2$  force size), whereas in reality there must be an assessment of a potential target's capability or performance rate. Models of the present type, and their offspring and siblings, should be of use for evaluating different proposed targetting strategies or doctrine.

$\underline{t}$	$\underline{R_a(t)}$				$\underline{B_a(t)}$			
	(A-I-C)	(C-A-I)	(A-I-C)	(A-I-C)	(A-I-C)	(A-I-C)	(A-I-C)	(C-A-I)
	$\underline{c_R=1,(1)}$	$\underline{c_R=1,(1)}$	$\underline{c_R=1,(2)}$	$\underline{c_R=1,(2)}$	$\underline{c_R=1,(1)}$	$\underline{c_R=1,(1)}$	$\underline{c_R=1,(2)}$	$\underline{c_R=1,(2)}$
0	0	0	0	0	0	0	0	0
1	20	20	20	20	10	10	10	10
2	37	38	39	39	18	18	19	19
3	52	53	57	57	24	24	27	26
4	64	64	74	75	30	30	34	32
5	75	72	91	91	34	34	41	36
6	81	77	87	88	37	37	47	39
7	77	79	82	84	40	40	52	41
8	73	80	77	80	72	72	57	42
9	69	79	71	76	43	43	61	43
10	65	76	65	71	45	45	65	43
15	44	55	39	51	48	48	64	38
20	26	33	23	33	33	33	48	30
25	16	19	13	20	21	21	38	22
Col:								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

Fig. 6.2

$$c_R = c_B = 1, \quad \rho_C = \beta_C = 0.10$$

Various Priorities

Cases: Cols. 2 & 6

Cols. 3 & 7

Cols. 4 & 8

Cols. 5 & 9

(1): Strong Central Control

(2): Loose Central Control

PROGRAM LISTING MODEL I  
(DETERMINISTIC & STOCHASTIC)

: RWAR FORTRAN A NAVAL POSTGRADUATE SCHOOL

REAL BU(100),BA(100),BC(100),RCC(100),RA(100)	RWA00010
REAL KC,K,H,THUA,PHI,BO,RO,RHOUC,RHUA	RWA00020
INTEGER T,TO	RWA00030
REAL*8 TITLE(3,2)	RWA00040
DATA TITLE/'DETERMIN','STIC C3',' MODEL ','STOCHAST','IC C3 MD',	RWA00050
X 'DEL'/'	RWA00060
DATA IX/123456/, IP/0/	RWA00070
DATA GAVEN COMBAT MODEL	RWA00080
RHOUC=0.	RWA00090
RHOUA=0.	RWA00100
BU(1)=30.	RWA00110
BA(1)=0.	RWA00120
BC(1)=20.	RWA00130
RCC(1)=30.	RWA00140
RA(1)=120.	RWA00150
H=1.	RWA00160
TO=61	RWA00170
CONTINUE	RWA00180
WRITE(6,121)	RWA00190
READ(5,122) K,KC,RHOAA,RHOAC,S,THUA,PHI,ITYPE	RWA00200
IF(K.LT.0.) STOP	RWA00210
	RWA00220
WRITE(6,111) K,KC,RHOUC,RHOUA,RHOAA,RHOAC,PHI,H,THUA,S	RWA00230
WRITE(6,123) (TITLE(I,ITYPE),I=1,3)	RWA00240
WRITE(6,100)	RWA00250
RO=RCC(1) + RA(1)	RWA00260
BO=BA(1)+BU(1)+BC(1)	RWA00270
BUAO=BA(1)+BU(1)	RWA00280
	RWA00290
DO 10 T=1,TO	RWA00300
R1=-1.	RWA00310
IF(BU(T)+BA(T).NE.0.) R1=RA(T)/(BU(T)+BA(T))	RWA00320
R2=-1.	RWA00330
IF(BA(T).NE.0.) R2=RA(T)/BA(T)	RWA00340
R3=-1.	RWA00350
IF(RA(1).NE.0.) R3=(RA(T)-RA(1))/RA(1)	RWA00360
R4=-1.	RWA00370
IF(T.GT.1.AND.RA(T).NE.0.) R4=(RA(T)-RA(T-1))/RA(T-1)	RWA00380
IT1=T-1	RWA00390
WRITE(6,101)IT1,RCC(T),RA(T),BU(T),BA(T),BC(T),R1,R2,R3,R4	RWA00400
	RWA00410
CUAN=CUA(BC(T),BU(T),THUA,K)	RWA00420
RCUA=SQRT(CUAN)*GAUSS(IX,IP,ITYPE)	RWA00430
BU(T+1)=AMAX1(BC(T)-CUAN-RCUA,0.)	RWA00440
BA(T+1)=AMIN1(BA(T)+CUAN+RCUA,BUAO)	RWA00450
	RWA00460
CC=CCC(RCC(T),PHI,BC(T),BO)	RWA00470
BC(T+1)=AMAX1(BC(T)-CC+SQRT(CC)*GAUSS(IX,IP,ITYPE),0.)	RWA00480
	RWA00490
CALL ALCC(RCC(T),BU(T),BA(T),RHOUC,RHOAC,RA(T),RO,KC,AC1,AC2)	RWA00500
RCC(T+1)=AMAX1(RCC(T)-(AC1+AC2)+	RWA00510
X SQRT(AC1+AC2)*GAUSS(IX,IP,ITYPE),0.)	RWA00520
	RWA00530
CALL ALA(RA(T),BU(T),BA(T),RCC(T),RHOUA,RO,RHOAA,KC,AL1,AL2)	RWA00540
	RWA00550

RWA01  
RWA01  
RWA01



## PROGRAM LISTING MODEL II (DETERMINISTIC)

: WACT      FURTRAN   A   NAVAL POSTGRADUATE SCHOOL

```

REAL R(3,100),B(3,100),DRA,DBA
REAL KR,KB, RO(3),BE(3), CBR(3),CRB(3)
INTEGER NAME(3)/'A','I','C'/
INTEGER I,TJ,VR(3),VB(3), PR(3),PB(3)
REAL*8 TITLE(3)
DATA TITLE/'PRIORITY',' TARGETT','ED MODEL'/
DATA IX/123456/, IP/0/
GAVER COMBAT MODEL PRIORITY TARGETTED
R(1,1)=0.
R(2,1)=100.
R(3,1)=20.
B(1,1)=0.
B(2,1)=150.
B(3,1)=10.
VR(1),I=1,3 ARE PRIORITIES FOR A,I,C RESPE. SAME FOR VB(I)
VR(1)=1
VR(2)=2
VR(3)=3
VB(1)=1
VB(2)=2
VB(3)=3
TO=61

```

```

CONTINUE
WRITE(6,121)
READ(5,122) KR,KB,CR,CB,(RO(I),BE(I),I=1,3)
IF(KR.LT. 0.) STOP
DO 6 I=1,3
  PR(VR(I))=I
  PB(VB(I))=I
  CRB(I)=-1.
  CBR(I)=-1.
CONTINUE

BAIO=B(1,1)+B(2,1)
RAIO=R(1,1)+R(2,1)
DRA=-1.
DBA=-1.
WRITE(6,111) KR,KB,CR,CB,(RO(I),BE(I),I=1,3)
WRITE(6,127) (NAME(VB(I)),I=1,3),(NAME(VR(I)),I=1,3)
WRITE(6,123) (TITLE(I),I=1,3)
WRITE(6,100)

```

```

DO 10 T=1,TO
  IT1=T-1
  WRITE(6,101) IT1,(R(I,T),I=1,3),(B(I,T),I=1,3),
X (CBR(I),I=1,3),(CRB(I),I=1,3),DRA,DBA

```

```

CRB(1)=AMIN1( B(PB(1),T), R(1,T) )
CRB(2)=AMAX1(0., AMIN1(B(PB(2),T),R(1,T)-CRB(1)) )
CRB(3)=AMAX1(0., AMIN1(B(PB(3),T),R(1,T)-CRB(1)-CRB(2)) )

```

```

CBR(1)=AMIN1( R(PR(1),T), B(1,T) )
CBR(2)=AMAX1(0., AMIN1(R(PR(2),T),B(1,T)-CBR(1)) )
CBR(3)=AMAX1(0., AMIN1(R(PR(3),T),B(1,T)-CBR(1)-CBR(2)) )

```

```

WAC00010
WAC00020
WAC00030
WAC00040
WAC00050
WAC00060
WAC00070
WAC00080
WAC00090
WAC00100
WAC00110
WAC00120
WAC00130
WAC00140
WAC00150
WAC00160
WAC00170
WAC00180
WAC00190
WAC00200
WAC00210
WAC00220
WAC00230
WAC00240
WAC00250
WAC00260
WAC00270
WAC00280
WAC00290
WAC00300
WAC00310
WAC00320
WAC00330
WAC00340
WAC00350
WAC00360
WAC00370
WAC00380
WAC00390
WAC00400
WAC00410
WAC00420
WAC00430
WAC00440
WAC00450
WAC00460
WAC00470
WAC00480
WAC00490
WAC00500
WAC00510
WAC00520
WAC00530
WAC00540
WAC00550

```



[illegible]

## REFERENCES

- Arnold, L. (1974) Stochastic Differential Equation: Theory and Application. Wiley-Interscience. John Wiley and Sons. New York.
- Bonder, S., and Farrell, R. (1970). "Development of models for defense systems planning." Systems Reserach Lab., Dept. of Industrial Eng., University of Michigan.
- Gaver, D.P. and Tonguç K. (1979) "Modelling the influnece of information on the progress of combat." MIT/ONR Symposium on Command & Control. Naval Postgraduate School.
- Lechoczky, J.P. and Perla, P.P. (1978). "Approximation technique and optimal decision making for stochastic Lanchester models." Carnegie-Mellon Statistic Dept. Technical Report 137.
- Taylor, J. (1979). "Attrition modelling," pp. 139-183 in Operational Research Games for Defense, ed. by Huber, Niemeyer, and Hofmann, R. Oldenburg, Verlag, Munich. (This paper contains many other useful references.)
- Tonguç K. (1979). Modelling the effect on information on conflict outcome. Naval Postgraduate School M.S. Thesis, Operations Research Department.

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